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The R_{uds} value in the vicinity of $\psi(3770)$ state

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ABSTRACT

The anomalous line shape of the $\psi(3770)$ state has resulted in some difficulty in the determination of the R value for the continuum light hadron production in the resonance energy range. We parameterize the asymmetric line shape using a Fano-type formula and extract the R_{uds} value to be 2.156 ± 0.022 from the data of BESIII Collaboration in the energy region between 3.650 and 3.872 GeV. The small discrepancy between experiment and theory is removed. The cross sections of the $e^+e^- \to hadrons$ with continuum light hadron production subtracted are given and compared to the data of the $e^+e^- \to D\bar{D}$ reaction.

1. Introduction

The cross section of the $e^+e^- \to hadrons$ is one of the most fundamental observables in Quantum Chromodynamics (QCD). The final hadrons are produced via quark–antiquark pair proceeded from a virtual photon by initial-state electron–positron annihilation. Instead of the cross section for inclusive hadron production, the hadronic R-ratio $R(s)$ is often used owning to its simplicity on both experimental and theoretical sides,

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad , \tag{1}$$

where $\sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/3s$ is the photon-mediated lowest order muon pair production cross section with s being the squared center-of-mas (c.m.) energy and α the electromagnetic coupling constant. If no resonances are present, the $R(s)$ values solely from the continuum hadrons are well known to be $3\sum_f Q_f^2$ in the lowest order approximation, with f being quark flavors and Q_f the corresponding quark charge. The higher order corrections from the finite quark masses and the gluonic emission could be calculated by perturbative QCD (pQCD) [1–3]. So the measurement of $R(s)$ is important for testing the validity of both pQCD calculation and hadronic vacuum polarization correction.

The $R(s)$ for the continuum light hadron (containing u, d and s quarks) production, denoted as R_{uds} in this letter, is usually used to test the validity of the pQCD calculation in relatively low energy region. Recently, there are several precise measurements of the R_{uds} near the $D\bar{D}$ threshold by BES Collaboration [4–7]. The R_{uds} value below the $D\bar{D}$ threshold is not affected by the first $D\bar{D}$ open charm resonance $\psi(3770)$ and therefore determination of R_{uds} is very simple in this case. The R_{uds} in the energy region from 3.650 to 3.732 GeV is determined to be $R_{uds} = 2.141 \pm 0.025 \pm 0.085$ [4], which is in good agreement with $R_{uds}^{pQCD} = 2.15$ predicted by pQCD [1–3]. However, the R_{uds} value above the open charm threshold is overlapped with many resonances. The obtained value varies widely among different fits and they have small discrepancy with the pQCD values. It is extracted to be $R_{uds} = 2.262 \pm 0.054 \pm 0.109$ in the energy region from 3.660 to 3.872 GeV [5] and to be $R_{uds} = 2.121 \pm 0.023 \pm 0.084$ from 3.650 to 3.872 GeV [6]. The central values of R_{uds} have varied in different fits, not because of the uncertainties of the data. Instead, the reason is that they are sensitive to the treatment of resonances in the $\psi(3770)$ region.

In order to accurately extract the R_{uds} in the region of $\psi(3770)$ resonance, the anomalous line shape of the $\psi(3770)$ state should be treated carefully and a more reliable method is needed. It has been found at very beginning that the total cross sections of $e^+e^- \to hadrons$ in the energy range between 3.700 and 3.872 GeV could not be described well with only one Breit–Wigner (BW) resonance even using the energy-dependent width of the $\psi(3770)$

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[4–7]. In the analysis of the BES Collaboration, the Flatté formula for the energy-dependent width is usually used in their fits to data,

$$\Gamma_{D\bar{D}}(s) \propto \frac{p^3_{0,\pm}(s)}{s(1+r^2 p^2_{0,\pm}(s))} \quad , \tag{2}$$

with $p_{0,\pm}(s) = \sqrt{s/4 - m^2_{D^{0,\pm}}}$ being the final D-meson momentum in the c.m. system and $r \sim 1.0$ fm the interaction radius of the $c\bar{c}$. The BW resonance with the width in Eq. (2) could give an asymmetric line shape of the $\psi(3770)$ state, but does not describe well the dip around 3.82 GeV. This is confirmed by the inclusive measurements of the $e^+e^- \to D\bar{D}$, D^+D^-, $D^0\bar{D}^0$ reactions in the similar c.m. energy region [8,9], where the asymmetric line shape with a dip behind the peak of the $\psi(3770)$ state is also found. These data from different measurements seem to have exclude the possibility that the dip is simply the statistical fluctuation.

The line shape of the $\psi(3770)$ state has inspired a lot of interesting theoretical efforts [10–18]. The main decay channel of $\psi(3770)$ resonance is shown to be $D\bar{D}$ by both BES and CLEO Collaborations, though the specific decay ratio is still under discussion [19–25]. The rescattering of final $D\bar{D}$ is found to be not enough to account for the line shape deviation [10]. Now it is uncovered to be the consequence of the interference between the $\psi(3770)$ resonance and the continuum background from the $\psi(2S)$ contribution [11–16]. Its implication to the nature of $\psi(3770)$ state is also investigated in the Fano mechanism [17,18]. In the Fano theory, the asymmetric line shape of states is produced by the interference of continuum and resonance, which gives rise to a general physical phenomenon in many quantum systems, e.g. the nuclear, atomic, condensed matter physics and molecular spectroscopy. Although the underlying physics of the $\psi(3770)$ state is still waiting for further exploration [17,18], the Fano-type formula provides an appropriate and simple parameterized expression for describing the anomalous line shape in the cross sections of the $e^+e^- \to hadrons$ and $D\bar{D}$. In this letter, we will use this formula to extract the R_{uds} value from experimental data reported by BES Collaboration in the energy region between 3.650 and 3.872 GeV [4].

2. Method and result

The theoretical $R_{uds(c)+\psi'}(s)$ contains the contributions from continuum light hadron production $R_{uds}(s)$, the continuum $c\bar{c}$ production $R_{(c)}(s)$, and the bare ψ' resonance production (here and below, the $\psi(3770)$ is denoted as ψ' for short), which is written as

$$R^{th}_{uds(c)+\psi'}(s) = R_{uds}(s) + R_{(c)}(s) + \frac{\sigma(e^+e^- \to \psi' \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad , \tag{3}$$

with $R_{(c)}(s) = f_{(c)} p^3_{0,\pm}/E^3_{0,\pm}$ in BESIII's fit [4]. The $\sigma(e^+e^- \to \psi' \to hadrons)$ is the hadrons production cross section through the bare ψ' resonance in e^+e^- annihilation, and it could be written in terms of the form factor $F_{\psi'}(s)$ in the following way:

$$\sigma(e^+e^- \to \psi' \to hadrons) = \frac{8\pi\alpha^2}{3s^{5/2}}[p^3_0(s) + p^3_\pm(s)]|F_{\psi'}(s)|^2 \quad , \tag{4}$$

where besides the factors from phase space, the bare ψ' form factor $F_{\psi'}(s)$ would be taken as the BW form:

$$F_{\psi'}(s) = \frac{g_{\psi' D\bar{D}} g_{\psi'\gamma}}{s - m^2_{\psi'} + i m_{\psi'}\Gamma_{\psi'}(s)} \quad , \tag{5}$$

where $g_{\psi' D\bar{D}}$ and $g_{\psi'\gamma}$ are the coupling constants of the ψ' to the $D\bar{D}$ and photon, respectively. Experiments indicate that the dominated decay channel of ψ' resonance is the $D\bar{D}$. Hence, we may use the energy dependent width

$$\Gamma_{\psi'}(s) = \Gamma_{D\bar{D}} + \Gamma_{nonD\bar{D}} = g^2_{\psi' D\bar{D}} \frac{p^3_0(s) + p^3_\pm(s)}{6\pi s} + \Gamma_{nonD\bar{D}} \quad , \tag{6}$$

or an improved parameterization of $\Gamma_{D\bar{D}}$ in Eq. (2). However, as we have addressed in Section 1, Eq. (5) is enough to describe the asymmetric line shape of the $\psi(3770)$ state, but does not describe well the dip around 3.82 GeV. The main weakness of above treatment is the totally separation of the continuum and resonant $D\bar{D}$ production in Eq. (3), but in fact, they are convoluted to each other in a sophisticated way. This is justified by the BESIII's fit results with the lower limit of $f_{(c)} \sim 0$ within the uncertainties. Keeping this in mind, we correct Eq. (3) as,

$$R^{th}_{uds(c)+\psi'}(s) = R_{uds}(s) + \frac{\sigma(e^+e^- \to (c\bar{c}) + \psi' \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad . \tag{7}$$

The $\sigma(e^+e^- \to (c\bar{c}) + \psi' \to hadrons)$ is the hadrons production cross section through the continuum $c\bar{c}$ and the ψ' resonance in e^+e^- annihilation, which should be alike to Eq. (4):

$$\sigma(e^+e^- \to (c\bar{c}) + \psi' \to hadrons) = \frac{8\pi\alpha^2}{3s^{5/2}}[p^3_0(s) + p^3_\pm(s)]|F_{(c)+\psi'}(s)|^2 \quad . \tag{8}$$

Instead of Eq. (5), the Fano-type form factor including the interference between resonance and continuum background could be written as [17,18,26,27]

$$|F_{(c)+\psi'}(s)|^2 = |g_{\psi' D\bar{D}} g_{\psi'\gamma} F_{(c)}|^2 \frac{|q+\varepsilon|^2}{1+\varepsilon^2} \quad , \tag{9}$$

with $\varepsilon = (-s + m^2_{\psi'})/(m_{\psi'}\Gamma_{\psi'})$. In the present context, the Fano line shape parameter q characterizes the relative transition strength from the ψ' state into the $D\bar{D}$ continuum, and can be related to the electromagnetic transition probability of the ψ' state. The q is an energy dependent variable in the original formula but regraded as a constant in the present limited energy range of interest. The factor $F_{(c)}$ comes from the non-resonant background possibly associating with either the direct $\gamma^* \to D\bar{D}$ transition or the nearby $\psi(2S)$ or other unknown charmonium states. Because the background contribution would be different in various channels, the line shapes of the ψ' would not be the same in other channels, e.g. $\psi' \to p\bar{p}$ [29] and $p\bar{p}\pi^0$ [30]. This is obviously true for other hadron states as well. However, here we do not dig into this issue and parameterize $F_{(c)}$ as $F_{(c)}(s) = 1/(s - m^2_{bg} + i m_{bg}\Gamma_{bg})$ for simplicity. It should be pointed out that the $F_{(c)+\psi'}(s)$ could be parameterized in other format, e.g. the coupled-channel models [13,14,18], however at the price of more complex.

The measured $R^{ex}_{uds(c)+\psi'}$ values versus c.m. energies are taken from BESIII's report [4], as shown in Fig. 1 with only statistical error bars. We fit these $R^{ex}_{uds(c)+\psi'}(s)$ data at each energy point to the theoretical formula described above using the least squares fitting method. The objective function of the least squares to be minimized in the fit is defined as

$$\chi^2 = \sum_{i=1}^{68} \left(\frac{R^{ex}_{uds(c)+\psi'}(s_i) - R^{th}_{uds(c)+\psi'}(s_i)}{\Delta R^{ex}_{uds(c)+\psi'}(s_i)} \right)^2 \quad , \tag{10}$$

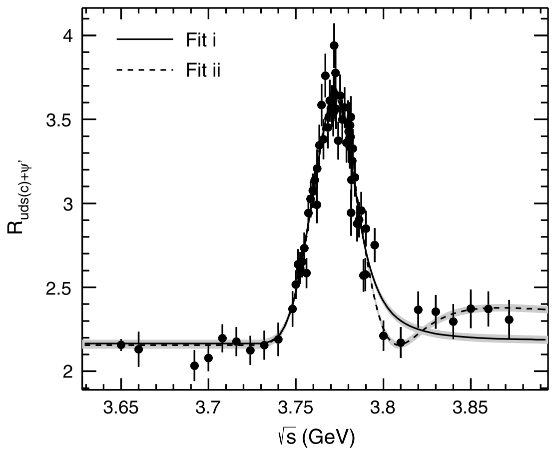

Fig. 1. The $R_{uds(c)+\psi'}(s)$ at different c.m. energies. The curves are the fits to the data with (solid line) and without (dashed line) $g_{\psi'\gamma}$ fixed. The bands reflect the variation of the fitted χ^2/ndf within 1σ. The data are measured by BESIII Collaboration [4].

where $R^{ex}_{uds(c)+\psi'}(s_i)$ is the measured value with the statistical error $\Delta R^{ex}_{uds(c)+\psi'}(s_i)$ at the squared c.m. energy s_i, and $R^{th}_{uds(c)+\psi'}(s_i)$ is the corresponding theoretical value calculated by Eq. (7).

In the considered narrow energy range, the R_{uds} could be viewed as a constant, independent of the energy. The width $\Gamma_{nonD\bar{D}}$ in Eq. (6) tends to be in the range of 0–5 MeV with large uncertainty in various fitting strategies. We do not include it into the following fits (i.e. $\Gamma_{nonD\bar{D}} = 0$ MeV). Therefore we have seven free parameters (R_{uds}, q, $m_{\psi'}$, $g_{\psi'D\bar{D}}$, $g_{\psi'\gamma}$, m_{bg} and Γ_{bg}) in total. As a reference, the $g_{\psi'\gamma}$ could be guided by the leptonic width $\Gamma_{\psi'e^+e^-} = 4\pi\alpha^2 g^2_{\psi'\gamma}/3m^3_{\psi'}$. However, it should be very cautious about this relation because the $g_{\psi'\gamma}$ in above formula would be connected to the leptonic width in more complex way. The real relation is waiting for more theoretical effort so the detailed comparison of our parameters to the Breit–Wigner parameters is suspended.

Here we perform two separate fits to the data. One of them (Fit i) is to fix $g_{\psi'\gamma} = 0.2523$ GeV^2 by the $\Gamma_{\psi'e^+e^-} = 0.262$ keV in Particle Data Group [28], and the other (Fit ii) is to let it being a free parameter. The curves in Fig. 1 show these fits with 1σ band of χ^2/ndf, where the solid line is for Fit i and the dashed line is for Fit ii. The corresponding fitted parameters are shown in Table 1, where the errors are only statistical ones. The achieving $\chi^2/ndf =$ 1.38 and 1.23 respectively for Fit i and Fit ii are obviously smaller than BESIII's result $\chi^2/ndf = 94/61 = 1.54$ [6]. Particularly, Fit ii gives a dip around 3.82 GeV, which describes the data perfectly well. If only the data above 3.80 GeV are used as the guideline of fit quality, the $\chi^2/N(\sqrt{s} \geq 3.80\ \text{GeV}) = 2.88/8 = 0.36$ for Fit ii is much better than $16.5/8 = 2.06$ for Fit i. The Fit ii gives $R_{uds} =$ 2.156 ± 0.022, whose central value is in excellent agreement with the prediction of pQCD [2] and can directly be used to evaluate the strong coupling constant $\alpha(s)$ at the energy scale of around 3 GeV. The error of the R_{uds} is on the same level of BESIII's result, and it still has some room for the improvement of the fit quality. This is probably due to the big uncertainties of the data, of which systematic errors are around the same scale of the statistical errors and not included here yet [4]. As a reference, the χ^2/ndf could be close to 1.0 within above formula in a similar fit to the data of $e^+e^- \to D\bar{D}$ reactions [18].

The parameter q has big error in Fit ii. As depicted in Eq. (9), its value largely rests on the position of the dip in the line shape, which is, however, has large uncertainty. Thus it could deduce that the uncertainty of q in Fit ii comes from the coincidence of the fitted $m_{\psi'}$ and the dip position. In addition, the sign of q varies in Fit i and Fit ii. This is due to the fitted $m_{\psi'}$ in these two fits lie on the opposite sides of the dip position. It is suggested to measure more data points near dip position in order to effectively reduce the uncertainties of the fitted parameters.

Table 1
Fitted parameters and achieving χ^2/ndf in Fit i and Fit ii, see text for details.

	Fit i	Fit ii
R_{uds}	2.165 ± 0.024	2.156 ± 0.022
q	1.58 ± 0.31	-0.19 ± 0.21
$m_{\psi'}$ (MeV)	3784.4 ± 2.7	3816.0 ± 13.9
$g_{\psi'D\bar{D}}$	14.0 ± 0.8	14.1 ± 3.4
$g_{\psi'\gamma}$ (GeV^2)	0.2523 (fixed)	0.417 ± 0.048
m_{bg} (MeV)	3753.6 ± 4.6	3767.4 ± 2.6
Γ_{bg} (MeV)	37.9 ± 3.2	41.9 ± 6.0
χ^2/ndf	$85.52/62 = 1.38$	$74.94/61 = 1.23$

It is found that the fitted $m_{\psi'}$ in both Fit i and Fit ii is larger than the BW values in PDG [28], even considering their big uncertainties. Our obtained values should be treated as bare mass of the ψ' as argued in Ref. [17]. Moreover, the fitted $m_{\psi'}$ depends on the way of dealing with the background term F_{bg}. Nonetheless, the corresponding dressed mass would be close to the PDG values, but more sophisticated models are involved to extract its value [18]. The width $\Gamma_{\psi'} \sim 29$ MeV at the nominal mass $m_{\psi'} = 3.773$ GeV calculated with the obtained $g_{\psi'D\bar{D}}$ is consistent with the BW values in PDG. The obtained $g_{\psi'\gamma}$ in Fit ii is bigger than that of the PDG value, and the corresponding leptonic width $\Gamma_{\psi'e^+e^-}$ is around 2.7 times bigger than that of the PDG if the usual relation is used. As shown in Eq. (9), the value of $g_{\psi'\gamma}$ is directly relevant to the form of F_{bg}. As pointed out above, the $g_{\psi'\gamma}$ would have a more intricate relation with the leptonic width in this format of the F_{bg}.

In our approach, the definition of resonances and background is clear, but they are coherently added together in a sophisticated way [11–16]. So the determination of the relation of the m_{bg} and Γ_{bg} to the parameters of $\psi(3686)$ state is postponed to later work. However, it is impossible to accurately determine the mass and width of the $\psi(3686)$ by the present data with relative big errors in the present framework. It should be addressed that the extracted R_{uds} is stable and reliable regardless of the uncertainties of these parameters, as long as the line shape of ψ' state is correctly reproduced.

Using the R_{uds} value extracted in Fit ii, we can obtain the cross sections of $e^+e^- \to (c\bar{c}) + \psi' \to hadrons$ by Eq. (7), as shown in Fig. 2. The data of the $e^+e^- \to D\bar{D}$ cross sections from BESIII Collaboration [5,9,22] are plotted in the same figure for comparison. The peak of $D\bar{D}$ production cross section is obviously smaller and narrower than that of *hadrons*, which hints at a non-zero $\Gamma_{nonD\bar{D}}$. We calculate the cross section of *hadrons* production to be 7.40 ± 0.69 (stat.) nb at $\sqrt{s} = 3774.2$ MeV, which is bigger than the recent CLEO result $\sigma(e^+e^- \to D\bar{D}) = 6.489 \pm 0.025 \pm 0.070$ at $\sqrt{s} = 3774 \pm 1$ MeV. However, they are still consistent with each other when both the statistical and systematic uncertainties are taken into account. Thus, the non-$D\bar{D}$ decay ratio of the ψ' is still waiting for more precise measurements.

3. Summary

In short summary, we have performed a renewed analysis of the measured R_{uds} value from BESIII Collaboration by treating the anomalous line shape of the $\psi(3770)$ resonance with a Fano-type formula. Our fitted results are better than those using a simple Breit–Wigner resonance with energy dependent width, mainly

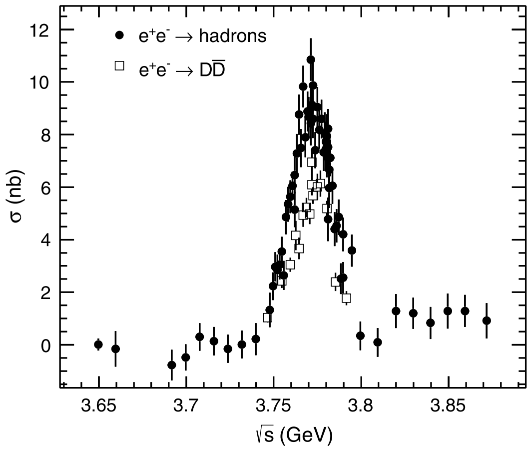

Fig. 2. Cross sections of the $e^+e^- \to (c\bar{c}) + \psi' \to hadrons$ reaction extracted from measured R ratios at different c.m. energies compared to that of the $e^+e^- \to D\bar{D}$ [9].

owning to the improvement on the description of the dip structure at about 3.82 GeV. In this sense, our parameterization would be more reasonable. The R_{uds} value is determined to be 2.156 ± 0.022 in the energy region between 3.650 and 3.872 GeV from the data of BESIII Collaboration. The central value is consistent with the pQCD calculation, although more precise measurements are needed to decrease its errors. Only after the errors of both data and fit method are reduced, we would have a trustable comparison between pQCD calculation and experiment. We also reliably extract the cross sections of the $e^+e^- \to hadrons$ without the continuum light hadron production, which would be beneficial to our understanding of the properties of the $\psi(3770)$ state.

Our prescription of fitting the asymmetry line shape of states is not only useful for pinning down the controversial decay ratios of $\psi(3770)$ state, but also meaningful for determining the R value in higher energy region where often has overlapped resonances. The proposed framework is easily extended to study other asymmetric line shapes of states and could be served as a simple fitting strategy to the experimental data.

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